BOND ANALYTICS

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Bond Valuation-Basics

- The basic components of valuing any asset are:
  - An estimate of the future cash flow stream from owning the asset
  - The required rate of return for each period based upon the riskiness of the asset
- The value is then found by discounting each cash flow by its respective discount rate and then summing the PV's (Basically the PV of an Uneven Cash Flow Stream)

What is a bond?

- A bond is basically a debt contract issued by a corporation or government entity.
- The buyer is lending the issuer an amount of money (the par value). The issuer agrees to pay interest at specified intervals (coupon payments) to the buyer, and return the par value at the end of the contract (the maturity date)

What makes a bond?

Par Value:
Initial issue amount
Coupon Payment:
Interest payments on the par value.
Coupon Rate:
The rate that determines the coupon payments.
Maturity Date:
The point in time when the par value and final coupon payment are made.
Embedded Options (Call and Put Provisions):
The issuer may be able to “call” the bond prior to its maturity.
Market Price:
The current price the bond is selling for in the market.

Bond cash flows….

- What component of a bond represents the future cash flows?

  Coupon Payment: The amount the holder of the bond receives in interest at the end of each specified period.

  The Par Value: The amount that will be repaid to the purchaser at the end of the debt agreement.

Value of the Bond

- Given

  r: The interest rate per period or return paid on assets of similar risk

  CP: The coupon payment

  MV: The Par Value (or Maturity Value)

  n: the number of periods until maturity

- The value of the bond is represented as:

$$V_{Bond} = \left[\sum_{t=1}^{n} \frac{CP_t}{(1+r)^t} \right] + \frac{MV}{(1+r)^n}$$

The rates of discount.

- So far we assumed that the interest rate is the same as the coupon rate. When this is true the value of the bond equals the par value.
- Are the two usually the same?

No, the discount rate should represent the current required return on assets of similar risk. This changes as the level of interest rates in the economy changes

Calculating Return

- Total Return (yearly) – The combined capital gains yield and interest (current) yield from holding the bond one year.
- Yield to Maturity – The yearly return if you purchase the bond today and hold it until maturity
- Yield to Call – The yearly return if you purchase the bond today and hold it until it is called.
- Yield to Put – the yearly return if you purchase the bond today and hold it until the put option is exercised.

Yield to Maturity

- Before we were looking for the “value” of the bond given a required rate of return.
- Now given the current market price we want to find the interest rate that makes the cash flows from the bond equal to its market price - this rate is known as the Yield to Maturity.
- The YTM is the return you earn IF you buy the bond today and hold it until maturity.

Calculating YTM

- Unfortunately calculating YTM is difficult:
- on excel use the Yield command
- =Yield(settlement, maturity, rate, price, redemption value, frequency, basis)

YTM and Risk

- The YTM will change as the level of interest rates in the economy change and as the risk associated with the firm and its projects change.
- The YTM is a representation of the probability of default and the current level of interest rates in the economy.

Promised or Expected Return

- You will earn the YTM if the bond does not default and you hold it to maturity.
- The expected return should encompass the chance of default, probability the bond is called or a put option is exercised, and the possibility of interest rate fluctuations.
- The YTM is only the expected return if the prob. of default is zero, the prob. of call or put is zero, and interest rates remain unchanged.

Yield to Call

- The yield to call is the yield paid on the bond assuming that a call option is exercised, given the current market price.
- It represents the yield you would earn if you bought the bond today and held it until the call option was exercised.

Yield to Worst

- After calculating all the possible Yields (yield to call, yield to put) the one with the lowest return is the termed the yield to worst.

Some points to consider

1) If the level of interest rates in the economy increases the bond price decreases and vice versa.
2) If $r > $Coupon rate the price of the bond is below the par value - it is selling at a discount.
3) If $r < $Coupon rate the price of the bond is above the par value - it is selling at a premium.
4) Keeping everything constant the value of the bond will move toward par value as it gets closer to maturity.

Complications

- Most bonds make payments every six months instead of each year.
- We have assumed that the next coupon payment is exactly 6 months away, often that is not the case. When the time frame is less than 6 months you need to account for interest over the shortened period.
- We have assumed that the interest rate is constant, some bonds pay a floating rate of interest.

Semiannual Compounding

- Most bonds make coupon payments twice a year, to account for this:
- Divide the annual coupon interest payment by 2.
- Multiply the number of periods by 2.
- Divide the annual interest rate by 2

Bond Price Volatility

- Assuming an option free bond, we have shown that the price and yield move in an opposite direction, however there are some important details:
  - Given similar bonds that differ only in maturity or coupon rate, The % price change associated with the same size change in yield will differ.
  - For a given bond the % price change associated with a small change in yield is the same regardless of whether the yield increases or decreases.

Bond Price Volatility continued

- For a given bond

  the % price change associated with a large increase in yield will not be the same as the % price change associated with the same size decrease in yield

  For a large change in yield the % price increase is greater than the % change decrease associated with the same size yield change.

Measuring Bond Price Volatility

- Price value of a basis point

    Measures the price change for a one basis point (.0001 or.01%) change in the yield of the bond.

- Yield value of a basis point

    Measures the change in the yield of the bond for a given price change.

- Duration

    Measures the price elasticity of the bond.

Duration

- Duration: Measures the sensitivity of the PV of a cash flow stream to a change in the discount rate.
- Keeping everything else constant the change in PV is greater:
  - The longer the time prior to receiving the cash flow
  - The larger the cash flow
  - (we just showed both of these)

Duration

- Calculation: Given the PV relationships, we need to weight the Cash Flows based on the time until they are received. In other words we are looking for a weighted maturity of the cash flows where the weight is a combination of timing and magnitude of the cash flows

Calculating Duration

- One way to measure the sensitivity of the price to a change in discount rate would be finding the price elasticity of the bond (the % change in price for a % change in the discount rate)

Duration Mathematics

- Macaulay Duration is the price elasticity of the bond (the % change in price for a percentage change in yield).
- Formally this would be:

$$D_{MAC} = \frac{\dfrac{\text{change in price}}{\text{original price}}}{\dfrac{\text{change in yield}}{\text{original yield}}} = \left(\frac{\text{Change in Price}}{\text{Change in Yield}}\right)\left(\frac{\text{Original yield}}{\text{Original price}}\right)$$

$$= \frac{\partial P}{\partial r}\frac{(1+\text{r})}{\text{P}}$$

Estimating Duration

- There are multiple methods for estimating the duration of a bond we will look at three different approaches.
  - Weighted Discounted Cash Flows (Macaulay)
  - Modified Duration
  - Averaging the price change

Duration Mathematics

- Taking the first derivative of the bond value equation with respect to the yield will produce the approximate price change for a small change in yield.

How much does the size of Bp change matter?

- What is the duration if we had assumed a 50 Bp change? Or a 100 Bp Change?
- Generally with a shorter maturity bond, the duration estimates will be very close, but for longer maturity bonds the duration estimates may differ by a small amount.

Duration Characteristics

Keeping other factors constant the duration of a bond will:

-Increase with the maturity of the bond

-Decrease with the coupon rate of the bond

-Will decrease if the interest rate is floating making the bond less sensitive to interest rate changes

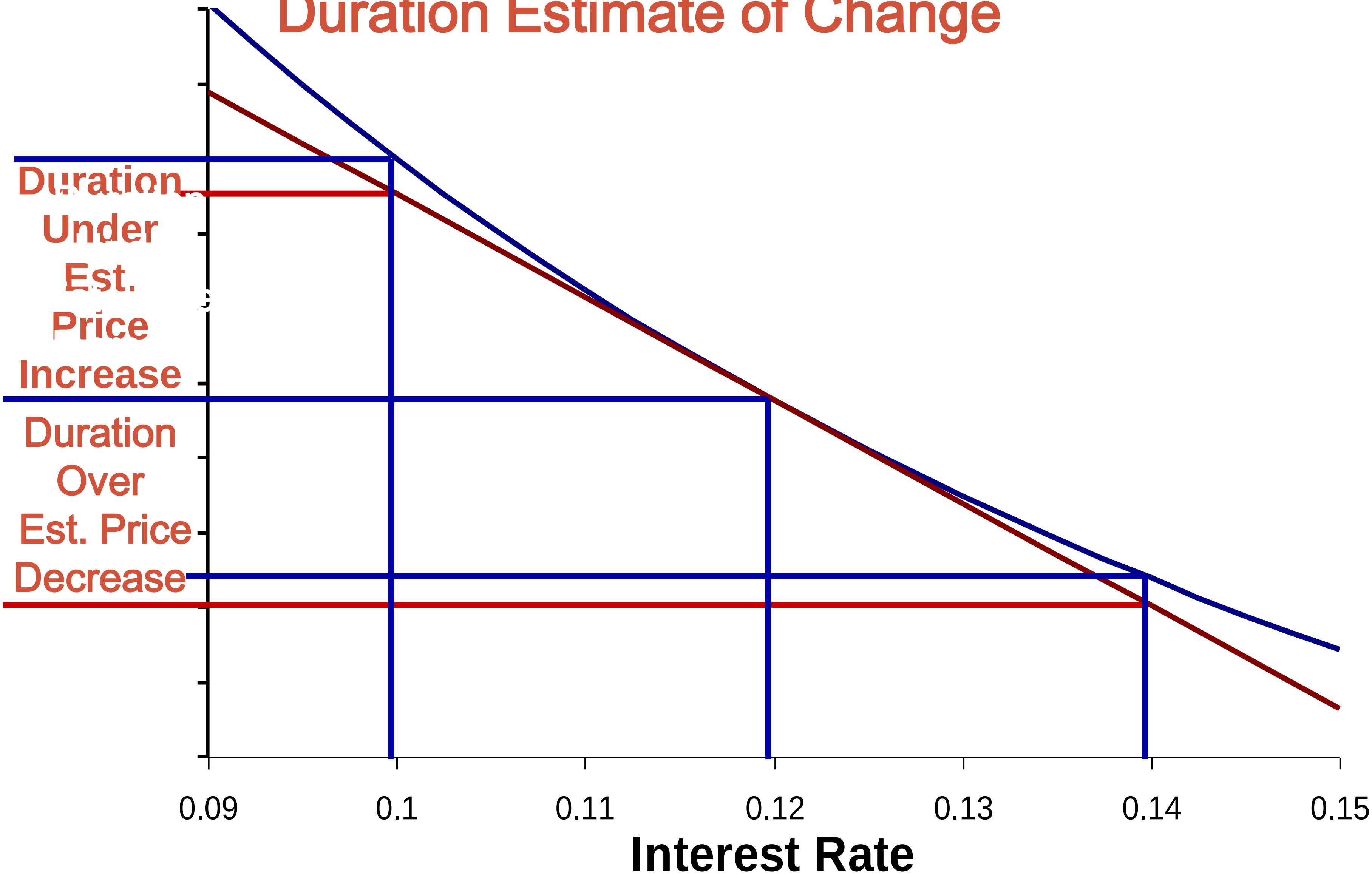
Effective Duration vs. Other Definitions

- Macaulay Duration and the most frequently used definition of modified duration assume that the cash flows do not change as the discount rate (yield) changes.
- Effective Duration accounts for an associated change in the cash flow, for example if a bond is called, or if mortgages are prepaid early.
- The linear approximation of Duration also implicitly assumes that cash flows can change. The value of the security should include any changes in the cash flows.

Duration and Convexity

- Using duration to estimate the price change implies that the change in price is the same size regardless of whether the price increased or decreased.
- The price yield relationship shows that this is not true.

Actual Price Change
Duration Estimate of Change



Convexity

- The amount of curvature in the yield price relationship is often referred to as the convexity.
- The curvature is measuring the change in the duration for a given change in yield (return).

Positive Convexity

- Generally: as the yield of the bond increases, the convexity of the bond decreases (positive convexity)
- This implies

  -As Yield increases, each successive price decline is less (there is a decline in the duration of the bond)

  -As Yield decreases, each successive price increase is greater (there is an increase in the duration of the bond)

Convexity

- Generally the following can also be said:
  - For a given return and maturity, the lower the coupon the greater the convexity
  - For a given return and modified duration, the lower the coupon the lower the convexity

Term Structure of Interest Rates

- How do demand and supply curves and shift of curves determine interest rate?
  - Theory of asset demand: bond market
  - Liquidity preference framework: money market
- Why interest rates are different?
  - Risk structure of interest rate
  - Term structure of interest rate

Expected return and risk

- Risk-return tradeoff.
- Then the ‘average’ of return, i.e. expected return is measured by ‘mean’ of R:

$$E(R) = \sum_N \pi_N R_N = \pi_1 R_1 + \pi_2 R_2 + \ldots + \pi_N R_N$$

risk is measured by variance or standard deviation

$$Var(R) = \sum_N \pi_N (R_N - E(R))^2$$

$$= \pi_1 (R_1 - E(R))^2 + \pi_2 (R_2 - E(R))^2 + \ldots + \pi_N (R_N - E(R))^2$$

$$\sigma(R) = \sqrt{Var(R)}$$

Term structure of interest rates

- Bonds with identical default risk, liquidity, and tax characteristics may **still have different interest rates** because the **time remaining to maturity** is different.
- Yield curve is **a plot of the yield on bonds with differing terms to maturity** but the same risk, liquidity and tax considerations.
- Shape of the yield curve:

1. **Usually upward-sloping**

   (long-term i > short-term i)

   sometimes 'inverted'

   (long-term i < short-term i)

2. **Flat** implies short- and long-term rates are similar

Facts about term structure of interest rates

1. Interest rates on bonds of different maturities **move together** over time.
2. When short-term interest rates are **low**, yield curves are more likely to have an **upward** slope; when short-term rates are **high**, yield curves are more likely to slope **downward** and be inverted – ‘mean reversion’.
3. Yield curves almost **always slope upward**.

Theories of the Term Structure

1. Expectations theory explains the first two facts but not the third.
2. Segmented markets theory explains fact three but not the first two.
3. Liquidity premium theory combines the two theories to explain all three facts

Expectations theory

- Assume buyers of bonds do not prefer bonds of one maturity over another -**perfect substitutes** ➔ expected returns of bonds with different maturity should equal.
- Conclusion:

  The interest rate on a long-term bond will equal **average** of the short-term interest rates that people **expect** to occur over the life of the long-term bond.

Example

- Suppose one-year interest rate over the next five years are expected to be: 5%, 6%, 7%, 8% and 9%
- Then, interest rate on the two-year bond:
  - (5% + 6%)/2 = 5. 5%
- Interest rate on the five-year bond:
  - (5% + 6% + 7% + 8% + 9%)/5 = 7%
- Interest rates on one to five-year bonds:
  - 5%, 5.5%, 6%, 6.5%, and 7%

Definitions – Spot Rates

- The n-period current spot rate of interest denoted r_n is the current interest rate (fixed today) for a loan (where the cash is borrowed now) to be repaid in n periods. Note: all spot rates are expressed in the form of an effective interest rate per year. In the example above, r_1, r_2, r_3, r_4, and r_5, in the previous slide, are all current spot rates of interest.
- Spot rates are only determined from the prices of zero-coupon bonds and are thus applicable for discounting cash flows that occur in a single time period. This differs from the more broad concept of yield to maturity that is, in effect, an average rate used to discount all the cash flows of a level coupon bond.

Definitions – Forward Rates

- The one-period forward rate of interest denoted f_n is the interest rate (fixed today) for a one period loan to be repaid at some future time period, n.
- I.e., the money is borrowed in period n-1 and repaid in period n.

Forward Rates

- To calculate a forward rate, the following equation is useful:

$$1 + f_n = (1+r_n)^n / (1+r_{n-1})^{n-1}$$

  - where f_n is the one period forward rate for a loan repaid in period n
    - (i.e., borrowed in period n-1 and repaid in period n)
  - Calculate f_2 given r_1=8% and r_2=9%
  - Calculate f_3 given r_3=9.5%

Definitions – Future Spot Rates

- Current spot rates are observable today and can be contracted today.
- A future spot rate will be the rate for a loan obtained in the future and repaid in a later period. Unlike forward rates, future spot rates will not be fixed (or contracted) until the future time period when the loan begins (forward rates can be locked in today).
- Thus we do not currently know what will happen to future spot rates of interest. However, if we understand the theories of the term structure, we can make informed predictions or expectations about future spot rates.
- We denote our current expectation of the future spot rate as follows: $E[{}_{n-t}r_n]$ is the expected future spot rate of interest for a loan repaid in period n and borrowed in period n-t.

Liquidity-Preference Hypothesis

- Empirical evidence seems to suggest that investors have relatively short time horizons for bond investments. Thus, since they are risk averse, they will require a premium to invest in longer term bonds.
- The Liquidity-Preference Hypothesis states that longer term loans have a liquidity premium built into their interest rates and thus calculated forward rates will incorporate the liquidity premium and will overstate the expected future one-period spot rates.

Segmented markets theory

- Assume: bonds of different maturities are **not** substitutes, investors **have preferences** for bonds of one maturity over another.
- If investors generally **prefer bonds with shorter maturities that have less interest-rate risk**, then this explains why yield curves usually slope upward.

Thank you !